Sweeping The RF Dipole Frequency Through The Resonance

-Comparision of a simple model with the experimental results-

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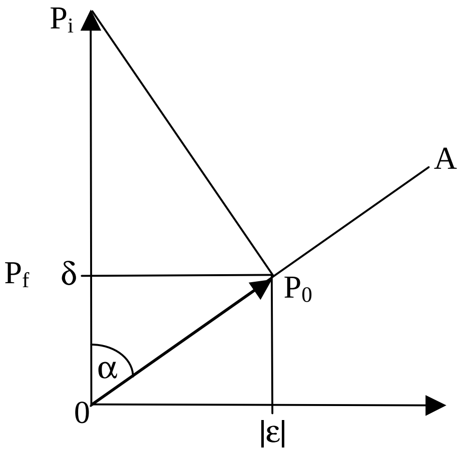
A. The Model



If the rf dipole is switched on with a strength $|\varepsilon|$ the initial vertical polarization P_i will change to P_0 which is the projection of P_i onto the axis $\overline{OA}$ determined by the resonance strength and the distance δ to the resonance. In the case of the experiment the distance is given by

$$\delta = \gamma_0 |G| - 1 + q$$

where $q = \dfrac{f_{RF}}{f_0}$ is the ratio of the rf dipole frequency to the revolution frequency. Turning off the rf dipole will the result in the final vertical polarization P_f.
The final polarization is found from the figure:

$$P_0 = P_i \cos\alpha \; and \; P_f = P_0 \cos\alpha$$

from which $P_f = P_i \cos^2\alpha$ follows. On the other hand the figure shows that $\cos\alpha = \dfrac{\delta}{\sqrt{\delta^2 + |\varepsilon|^2}}$ so that the final polarization after switching off the rf dipole is given by

$$P_f(\delta) = P_i \frac{\delta^2}{\delta^2 + |\varepsilon|^2}. \qquad (1)$$

Thus the final polarization is zero for $\delta = 0$ and $P_f = P_i \; for |\delta| \to \infty$. The full width at half maximum (FWHM) is $FWHM = \sqrt{2}\,|\varepsilon|$.
In general the distance to the resonance depends on momentum through

$$\delta(p) = \gamma(p)|G| - 1 + \frac{f_{RF}}{f(p)}.$$

Thus the final polarization P_f is found by averaging $P_f(\delta(p))$ with the momentum distribution $\psi(p)$ which is assumed to be normalized to one:

$$P_f = \int_{-\infty}^{\infty} P_f(\delta(p))\,\psi(p)dp \qquad (2)$$

We now make the assumption that $\delta(p)$ can be linearized in a region around the mean momentum p_0. The linearization is

$$\delta(p) = A(q) + B(q)\cdot(p - p_0)$$

with

$$A(q) = \gamma_0\,|G| - 1 + q$$

$$B(q) = |G|\frac{c}{E_r}\frac{\frac{p_0 c}{E_r}}{\sqrt{1+\left(\frac{p_0 c}{E_r}\right)^2}} - q\frac{\eta}{p_0}.$$

Here, $q = \frac{f_{RF}}{f_0}$ and $\eta = \frac{1}{{\gamma_0}^2} - \frac{1}{{\gamma_{tr}}^2}$ with $\gamma_0 = \gamma(p_0)$. E_r is the (deuteron´s) rest energy.

For the momentum distribution we now assume a Gaussian centered around p_0

$$\psi(p) = \frac{1}{\sigma\sqrt{2\pi}}\,e^{-\frac{1}{2}\left(\frac{p-p_0}{\sigma}\right)^2}.$$

From eq. (2) the final polarization $P_f(q)$ as a function of the ratio q is then found from

$$P_f(q) = \frac{P_i}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{(A(q)+B(q)\cdot(p-p_0))^2}{(A(q)+B(q)\cdot(p-p_0))^2+|\varepsilon|^2}\,e^{-\frac{1}{2}\left(\frac{p-p_0}{\sigma}\right)^2}dp. \qquad (3)$$

The integral in eq. (3) is solved numerically for COSY´s beam and machine parameters with a given resonance strength. An approximate solution is given below.

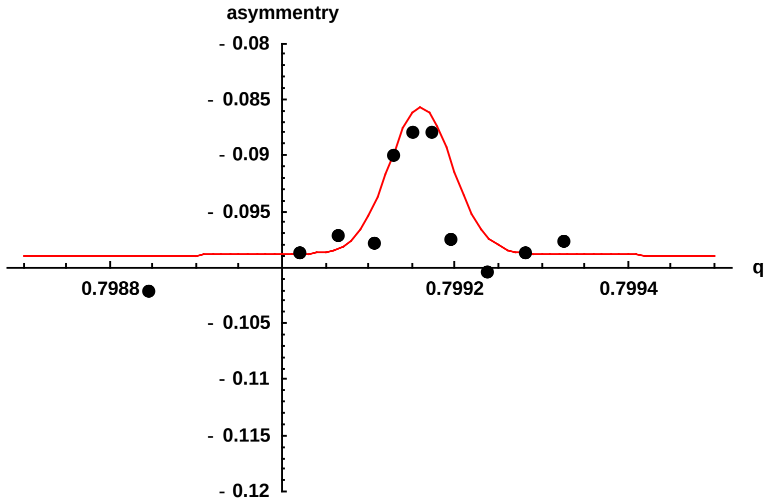
Note that even for $q = 1 - \gamma_0\,|G|$ the final polarization according to eq. (3) does not necessarily vanish for a beam with *finite* momentum spread. If the momentum spread becomes zero the beam distribution is a delta function and from eq. (3) one finds the relation eq. (1) so that at $q = 1 - \gamma_0\,|G|$ the final polarization will vanish. That is, eq. (1) holds only for a beam with zero momentum spread.

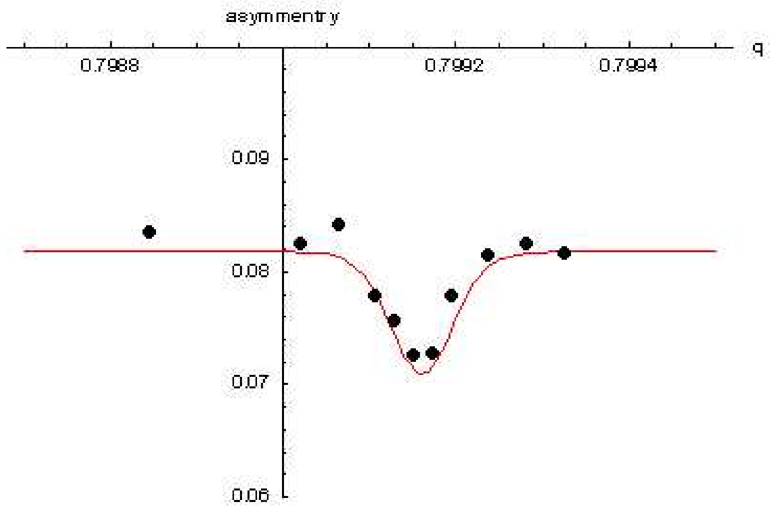
B. Results:

<u>Deuteron Run</u> <u>, week 6, 2003</u>

Mapping the $f_0\,(1-\gamma|G|)$ rf resonance:

The two figures show the measured asymmetries of two spin states (dots) when the rf frequency is mapped through the resonance. Here the asymmetry is plotted against the ratio $q = \frac{f_{rf}}{f_0}$. In comparison a fit according to eq. (3) to the data are shown (red curves).





The following values have been used:

momentum p_0	1850	MeV/c	
revolution frequency f_0	1.14728	MHz	measured
kinematic γ_0	1.40459		
frequency slip factor η	0.3		measured
relative momentum spread σ_p / p_0 (standard deviation)	$2.5 \cdot 10^{-4}$		
Resonance strength	$4.1 \cdot 10^{-6}$		calculated
G	- 0.1429878		

The figures show that the data are quite well described by the model. From the fit the center of the resonance f_R and the width σ_f (standard deviation) is determined:

$f_R = 916.86\,kHz$ and $\sigma_f = 43\,Hz \quad or \quad 4 \cdot \sigma_f = 172\,Hz$

If we use D. Prasuhns´ approximation $\sigma_f = f_0 \cdot |G| \cdot \frac{\gamma_0^2 - 1}{\gamma_0} \cdot \frac{\sigma_p}{p_0}$ (see SPIN@COSY) we find $\sigma_f = 28\,Hz \quad or \quad 4 \cdot \sigma_f = 112\,Hz$.

An numerical analysis of eq. (3) shows (and as the figures suggest) that the data can be indeed well described by a simple Gaussian (assuming the beam has a Gaussian momentum distribution):

$$P_f(q) = a \cdot (1 + b\,|\varepsilon|\,e^{-0.5\left(\frac{q - q_R}{\sigma_R}\right)^2}),$$

where a and b are fit parameters, $\sigma_R = \sigma_f / f_0 = |G| \cdot \frac{\gamma_0^2 - 1}{\gamma_0} \cdot \frac{\sigma_p}{p_0}$ and $q_R = 1 - \gamma_0 |G|$.

Proton Run, week 17, 2003

Mapping the $f_0\,(5 - \gamma|G|)$ rf resonance:

The same analysis will be done for the proton data if they are available. So far we estimated the width of the resonance according to eq. (3) and Dieter´s formula. The resonance strength was calculated and compared to the experiment (SPIN@COSY).

momentum p_0	1941	MeV/c	
revolution frequency f_0	1.471168	MHz	measured
kinematic γ_0	2.29772		
frequency slip factor η	0.03		measured
relative momentum spread σ_p / p_0 (standard deviation)	$2.5 \cdot 10^{-4}$		measured
Resonance strength	$2 \cdot 10^{-5}$		calculated, measured
G	1.79285		

We found

$f_R = 1295.4\,kHz$ and $\sigma_f = 1.23\,kHz \;\; or \;\; 4 \cdot \sigma_f = 4.92\,kHz$. Note that the width is significantly larger than that for the deuteron case, mainly due to the larger $|G|$ factor and γ_0.
Applying the approximate formula results in

$\sigma_f = 1.23\,kHz \;\; or \;\; 4 \cdot \sigma_f = 4.92\,kHz$,

i.e. the same values as derived by applying eq. (3).