Dieter Prasuhn
May 7th, 2003

SPIN@COSY

Calculation of Frequency and Frequency Span of the SPIN Flip Resonance

During the April 2003 Spin@COSY beam time with protons 2 questions came up:

1. The resonance frequency was found 10 kHz above the calculated resonance frequency.
2. The resonance width occurred to be $\pm$3 kHz wide in contrast to the deuteron beam time in February, where the resonance showed a width of only 100 Hz.

Ad 1.: In February for the **deuteron** beam time the resonance occurred at the right calculated frequency:

$$f_{resonance} = f_{revolution} \cdot (1 + \gamma \cdot G) \quad [1]$$

The flat top momentum for acceleration was 1850 MeV/c. A flat top revolution frequency of 1.14743 MHz was measured. The flat top orbit was well adjusted, so that dipole field and frequency in the flat top corresponded well to each other. With the relativistic Lorentz factor γ=1.4046 and the gyromagnetic anomaly G=-0.1429878 this results in the resonance frequency 0.916981 MHz. The measured resonance frequency of 0.91685 MHz is in good agreement with the calculated value.

In the case of the protons we had accelerated to 1941 MeV/c, measured a revolution frequency of 1.471168 MHz, but we measured a strong orbit deviation in the flat top which follows exactly the dispersion function. This indicates that frequency and dipole field do not correspond to each other, in this case the dipole field was too strong and the beam took the inner (shorter) way in the bending sections. Now, the frequency is calculated according to the flat top momentum and the nominal machine circumference. The beam particles are forced to the frequency by the cavity, the dipole is too strong, they take a shorter way, this means they have a smaller velocity, a smaller momentum. It was measured a resonance frequency of 1.305 MHz relative to the calculated 1.2953 MHz. But by a rearrangement of formula [1] one can now estimate from the measured frequencies the γ and the corresponding momentum.

$$f_{resonance} = f_{revolution} \cdot (5 - \gamma \cdot G) \quad [1a]$$

$$\gamma = \left(5 - \frac{f_{resonance}}{f_{revolution}}\right) \Big/ G \quad [2]$$

This results with $G_{protons}$=1.79285 in γ=2.29408, a momentum of 1937.189 MeV/c, which indicates a momentum deviation of $-1.96 \cdot 10^{-3}$ relative to the assumed momentum. And this is in good agreement with the measured closed orbit deviations and the dispersion function in COSY.

Ad 2.: A momentum spread of of thebeam results both in a spread of the revolution frequency and in a spread of the resonance frequency.

The revolution frequency spread is defined by the lattice function $\gamma_{transition}$:

$$\frac{\Delta f_{revolution}}{f_{revolution}} = \underbrace{\left(\frac{1}{\gamma^2} - \frac{1}{\gamma_{transition}{}^2}\right)}_{=\eta} \cdot \frac{\Delta p}{p_0} \qquad [3]$$

In the case of the deuterons with γ=1.4046 and $\gamma_{transition}$=2.2 we get η=0.3. An assumed momentum spread of $\Delta p/p=1*10^{-3}$ results in a spread of the revolution frequency of $(\Delta f/f)_{revolution}=3*10^{-4}$.

For the protons we measured η=0.03, so a momentum spread of $\Delta p/p=1*10^{-3}$ leads to a frequency spread of $(\Delta f/f)_{revolution}=3*10^{-5}$. At a revolution frequency of 1.4 MHz this means a frequency spread of $\Delta f_{revolution}$=40 Hz.

Now the spread of the resonance frequency:

The resonance frequency is defined by formula [1], now changed to the general

$$f_{resonance} = f_{revolution} \cdot (n \pm \gamma \cdot G) \qquad [1c]$$

The derivative ($\delta f_{resonance}/\delta\gamma$) leads to the equation

$$\frac{\delta f_{resonance}}{\delta\gamma} = \pm f_{revolution} \cdot G \qquad [4]$$

Thus the frequency spread of the resonance is given by:

$$\delta f_{resonance} = \pm f_{revolution} \cdot G \cdot \delta\gamma \qquad [5]$$

replacing the relativistic quantity $\delta\gamma$ by the momentum and momentum spread

$$\delta\gamma = \frac{\beta}{m_0} \cdot \delta p \qquad [6]$$

results in the equation

$$\delta f_{resonance} = \pm f_{revolution} \cdot G \cdot \frac{\beta}{m_0} \cdot \delta p \qquad [7]$$

Replacing the absolute momentum spread δp by the relative momentum spread ($\Delta p/p$) leads to the relation

$$\delta f_{resonance} = \pm f_{revolution} \cdot G \cdot \frac{\beta}{m_0} \cdot p_0 \cdot \left(\frac{\Delta p}{p_0} \right) \qquad [8]$$

In the case of ***deuterons*** with G= -0.1429878, a momentum of p_0=1850 MeV/c, a revolution frequency $f_{revolution}$=1.14743 MHz, β=0.702 a momentum spread of (Δp/p)=1*10^{-3} leads to a resonance frequency spread of

$$\delta f_{resonance} = 0.114 \text{ kHz}$$

which is in good agreement with the measured resonance width.

For the ***protons*** the large G=1.79285 directly leads to a larger frequency spread. With our data p_0=1941 MeV/c and the corresponding data we find for (δp/p)=1*10^{-3} a frequency resonance width of

$$\delta f_{resonance} = 4.912 \text{ kHz}$$

So, I believe that all measured data are quite well understandable.